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THE RICE INSTITUTE

A university of liberal and technical learning founded by William Marsh Rice in the City of Houston, Texas, and dedicated by him to the advancement of Letters, Science, and Art

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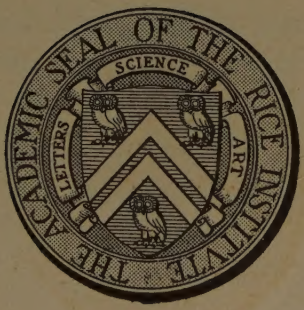

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I
RECENT ADVANCES IN STELLAR ASTRONOMY[1]

THE LIGHT OF THE STARS

THE study of the stars is the oldest of the sciences: yet it rests on the slenderest physical basis. Were our atmosphere always cloudy—instead of about half the time—human affairs, and most of human science, would go on very much the same. We could see nothing of the great universe beyond the clouds; yet the light of the Sun and Moon would penetrate them, and their attraction would still set the tides in motion, and reasoning from these data, we might deduce a good deal about the Sun and Moon. But the very existence of the stars would be unsuspected, for it is only by their light that we know them, and this light is far too feeble to pierce the clouds.

How feeble, indeed, it is, is one of the things which most of us fail to realize. The light even of Sirius—the brightest of the stars—is equal to that of a 25-candle lamp a mile away. Ordinary artificial light, such as is commonly used to read by, is a million times brighter, and full sunlight is ten thousand times stronger still. It is only because of the amazing range of adaptation of the human eye—which incomparably surpasses that of any instrument which has ever been constructed—that we can see the stars at

[1] A course of three lectures delivered by Professor Henry Norris Russell, Ph.D., of Princeton University, in the Physics Amphitheatre of the Rice Institute, January 26, 27, and 28, 1922.

all, and yet escape being blinded by the glare of day. But our eyes do better than merely to detect the almost infinitesimal quantities of energy which the stars send us: they give us very good reports regarding their relative amounts.

Almost from time immemorial, the stars have been classified according to their apparent brightness, and the system of describing this which is still used is well over two thousand years old. Since the days of Hipparchus the brightest of them have been called "stars of the first magnitude," those decidedly fainter, "of the second magnitude," and so on to the sixth magnitude—the faintest visible to the unaided eye. When accurate measures of starlight began to be made, about the middle of the nineteenth century, it was found that a 2nd magnitude star sent us about 2½ times as much light as one of the 3rd magnitude, the latter again 2½ times the light of a 4th magnitude star, and so on—a difference of five magnitudes corresponding to a light ratio of 100. With this principle as guide, the scale of magnitudes can be extended to the faintest stars visible with the telescope, which in the case of the hundred-inch instrument at Mount Wilson are of about the 21st magnitude, and backward to bright objects, for which the numbers expressing the magnitudes are negative, so that the magnitude of Venus is about —4; of the full Moon —12.5; and of the Sun —26.7.

We are not confined, however, to visual observations, when we measure the brightness of the stars. The intensity of the image on a photograph, if properly standardized, serves as well, and the photographic magnitudes which are thus derived give us much more than a mere check upon our previous results. The eye is sensitive primarily to green and yellow light (red and blue, for

equal energy, making but a relatively weak impression) so that our visual magnitudes are substantially measurements of the stars with yellow-green light. The ordinary plate on the other hand is influenced almost entirely by violet light.

It is, therefore, not surprising that the stellar magnitudes determined in the two ways often disagree. Red stars photograph relatively faint, and bluish ones bright. This has been known since the first stellar photographs were made, but its full importance was recognized much more recently. In this difference between the visual and photographic magnitudes we have a way of measuring the color of a star's light—for which reason it is called the *color-index*. The results are in excellent agreement with the direct estimates of color by visual observation, but have two great advantages—first, that we can assign to them a definite physical meaning, and second, that they are more trustworthy, for the eye is a rather poor judge of color in objects as faint as the stars. Indeed, by using an isochromatic plate, with a "color screen" to cut off the violet light and let through the green and yellow, we may get "photovisual" magnitudes which are substantially equivalent to the usual determinations, and in some cases better—notably for very faint stars, where the behavior of the eye has annoying peculiarities. By either method we find that sunlight is near the middle of the range of starlight colors. Stars like Vega, or, even more, those in the belt of Orion, are much bluer, while those like Arcturus, and especially Antares, are much redder. But the colors of the stars lie between definite limits, which may be described as corresponding to the emission of three times as much violet light, in proportion to the yellow, as in the Sun's case, and again one-third as much violet light. Between these limits all shades of color are found in nearly equal abundance;

but only about one star in a thousand is redder than the limit, and it is doubtful if any known star is bluer.[1]

Terrestrial sources of light, on the same scale, are found to be very red. All but a few of the whitest of them, such as the carbon arc or the gas-filled tungsten lamp, are redder than the reddest of the ordinary types of stars, and even the exceptions are much redder than the Sun. Our instinctive standards for estimating color, in fact, are quite different for the stars and for artificial light. A commonplace, though modern, illustration of this is the "daylight" incandescent lamp, which is carefully adjusted to give light of the color of sunlight, but, if seen among other lamps at night, looks blue.

So far, however, we have been talking only about the apparent brightness of the stars—the light which we get from them. What of their real brightness—the light which they send out? To determine this we must know the distances of the stars—and this opens up a long and difficult chapter of practical astronomy.

The first rational estimate of the distance of a star was reached from photometric considerations, and by no less distinguished a philosopher than Newton. He proved, first, that the stars do not shine by reflected sunlight—for they show no perceptible disks in the telescope and therefore they could not reflect light enough to account for their observed brightness. They must, therefore, be self-luminous like the Sun. Now Sirius looks rather fainter than Jupiter. From this fact, by ingenious but thoroughly sound reasoning, Newton showed that, if Sirius was really as bright as the Sun, its distance must be fully 100,000 times as great as that which separates the Earth and Sun. We know now that the actual distance is five times greater;

[1] Except the nuclei of certain nebulæ, which may not be ordinary stars.

but this determination, made by methods impracticable in Newton's day, detracts in no wise from the credit which admittedly he deserves for giving the first correct estimate of the order of magnitude of the remoteness of the stars.

It is well-nigh a century since Bessel determined the parallax and distance of 61 Cygni—applying principles essentially similar to those of the range-finders which are used with modern artillery. Slowly at first, then faster, the technique of observation improved, and now, by taking a dozen or so of photographs with one of our larger telescopes we can measure the distance of a star a million times as far away as the Sun, and be more likely than not to get it right within five per cent. There are not many stars as near as this—only a few score, among the millions which the telescope reveals—and for the remote ones the percentage error of our measures increases in proportion to their distance. But, up to ten million times the Sun's distance, the results of direct measures of parallax are still of value—particularly if we can take the average of three or four determinations by good observers.

To describe these enormous distances with convenience, we need a new unit of measurement. Astronomers are coming very generally to adopt the distance at which the parallax of a star would be one second of arc. The word *parsec* has been coined to denote this distance—which is 206,000 times the Earth's distance from the Sun, or, in common units, 19 millions of millions of miles. It may also be defined as equal to 3.26 light-years—the latter unit being the distance which light travels in a year. No known star lies within one parsec of the Sun. Two stellar systems are nearer than two parsecs, and there are probably three or four hundred nearer than ten parsecs—the majority

still awaiting observation. Our direct meaures of parallax are fairly good up to thirty, or even to fifty parsecs.

When we can get a star's distance, it is very easy to calculate its real brightness, and the range in the values is enormous. The faintest star so far known, is an eleventh magnitude star, discovered by Innes, which is a distant companion of the first magnitude star Alpha Centauri—two degrees away from it, but moving in the sky at the same rate, so that the two probably form one great system. Its parallax is slightly greater than that of the bright star, which makes it the nearest known object in the heavens (1.3 parsecs) and its real brightness is but one ten-thousandth part that of the Sun. It is only visible because it happens to be so near us. Alpha Centauri itself is just about as bright as the Sun, but our next nearest neighbor in space—another faint star discovered by Barnard—gives out about 1/2700 of the Sun's light.

Sirius (also a near neighbor, at 2.7 parsecs) is 25 times as bright as the Sun—and so our list might go on almost indefinitely, for there are more than a thousand stars for which direct measures of parallax have been made, but many of the brightest stars in the sky have parallaxes too small to measure in this way. All that our measures indicate is that their distances are greater than 50 parsecs, and probably than 100, and that their real brightness in some cases must be more than a thousand times the Sun's.

With such an enormous range of values to deal with, it is obviously desirable to have some simple method of expressing the real brightness of a star, as convenient as the stellar magnitude is for the apparent brightness. Kapteyn met the need by inventing the "absolute magnitude" which is simply the magnitude that the star would

appear to have if it were brought to a standard distance—which he chose as 10 parsecs. On this scale the absolute magnitude of the Sun is +4.8, that of a star 100 times fainter would be +9.8, that of Innes's star is +15; while a star 100 times brighter than the Sun would have an absolute magnitude of −0.2 (which is approximately the case for Arcturus or Vega). Still brighter stars, with negative absolute magnitudes, exist; but to obtain any accurate notion of their brightness we need some more powerful method of measuring distance.

One such method has developed from the discovery of a number of "moving clusters" of stars. All the members of each of these clusters are moving together in space—in parallel directions and at the same rate—though their proper motions in the sky appear, on account of perspective, to converge towards a definite vanishing point. When we know this point, and measure with the spectroscope the rate at which the stars of the cluster are receding from us (or perhaps approaching, in which case their proper motions diverge from a common point)—it becomes a simple matter of geometry to calculate the parallax and distance of every star in the cluster. The first of these clusters to be fully investigated (in a classical paper by Lewis Boss) is the Hyades group in the constellation Taurus, which contains about a hundred stars forming a roughly globular group about ten parsecs in diameter, with its centre forty parsecs from the Sun, and ranging in luminosity from fifty times the Sun's light to one-tenth of its brightness.

Of more immediate interest to us is a huge group in the southern sky studied by Kapteyn, and containing most of the brighter stars in Scorpius, Centaurus and the Southern Cross. The proper motions of these stars are small and

their distances great, averaging about 100 parsecs. Two of them are of the first magnitude, and much excel in luminosity any of the stars which we have previously mentioned, Beta Centauri possessing a luminosity 2,500 times the Sun's while Antares (Alpha Scorpii) exceeds the Sun's light a little more than three thousand fold. There are many other stars in this cluster which are several hundred times brighter than the Sun.

Another similar star group comprises most of the conspicuous stars in Orion, and many in Canis Major. This group is even farther off, but it is difficult to estimate its distance with precision, for it is moving almost straight away from us, so that the stars seem to stand almost still in the sky. The majority of the stars appear to be at distances of between 150 and 200 parsecs. Among these are the three stars of Orion's belt, which average about 4,500 times as bright as the Sun, and the still brighter Rigel, in the southern part of the constellation, which appears to give out some 13,000 times the Sun's light and be the brightest star at present known. Two other stars of the first magnitude, Canopus and Alpha Cygni, are also known to be exceedingly remote, but we cannot yet measure their distances. One or both may be as luminous as Rigel, or even brighter.

We have, therefore, among the stars a range in real luminosity of at least a hundred million fold. The physical interpretation of such great differences must obviously be of prime importance. But before we can discuss it intelligently we must analyze the light of the stars more thoroughly. We can do this best by photographing their spectra. For those visible to the naked eye this can be done in considerable detail; and the brighter telescopic stars, down to the ninth or tenth magnitude, are accessible

with lower dispersion. Years of study at Harvard, mainly by E. C. Pickering, Mrs. Fleming and Miss Cannon, have culminated in the classification of the spectra of more than 200,000 stars.

The familiar, but nevertheless amazing, result of these researches is that all but a minute fraction of these can be placed in one or another of six spectral classes, which, during the progress of the work, came, by the law of survival of the fittest in nomenclature, to be designated by the arbitrary letters B, A, F, G, K, and M. What is still more noteworthy is that these six classes grade into one another imperceptibly, so that they form parts of a single sequence (in the order named above). The transition from any type to the next always takes place through the same intermediate stages (illustrated, of course, by different stars) so that we may adopt the familiar decimal classification, and call a star half way between B and A, for example, B5A, or simply B5.

This linear sequence of types finds room for more than 99 per cent. of the stars. The exceptions can almost all be placed in three additional classes—denoted by the letters O, R and N—of which the first evidently belongs at the head of the list, before B, and is connected with it by intergrades, while the other two form a sort of side-chain branching from the main sequence near K.

Spectra near the head of this sequence show mainly the lines of the permanent gases—hydrogen, helium, oxygen and nitrogen. As we pass on from B to A the helium lines disappear, and lines of the metals come in —at first those lines which show in the laboratory only, or chiefly, in the spectrum of the electric spark. Farther on, in Class F, the arc lines of the metals appear, and in

Class G, (to which the Sun belongs) they are predominant, while the spark lines become weaker as do also the lines of hydrogen. This change continues into Class K, and the flame lines of the metals, (produced in a flame, or an electric furnace, at relatively low temperature) become conspicuous. Finally, in Class M, heavy absorption bands appear, which have been identified as due to the presence in the star's atmosphere of the vapor of titanium oxide. Another set of bands, produced by compounds of carbon, is prominent in Classes R and N.

What physical or chemical conditions are there behind this succession of types? Are the atmospheres of the various stars really as different in chemical composition as their spectra appear to indicate? This is a tempting hypothesis at first sight: but there are weighty objections to it.

First of all, the sequence of spectra is linear. In passing from a given type to those which closely resemble it, we have practically only two lines to follow—up or down the series. It seems an inevitable deduction from this that the spectrum of a star might be described in mathematical language, as a function of a single variable. Now the chemical constitution of an atmosphere is a function of many variables. For all that we know, the amounts of helium, oxygen, iron, hydrogen, and other elements in it may vary independently. If variations of this sort lay behind the spectral differences, there is no apparent reason why the lines of different elements should always show so definite a relation between their behavior—or, indeed, any conspicuous relation at all.

Moreover, we know of pairs of stars, so close that no telescope can separate them, whose duplicity is known only because one component of the system eclipses the

other at regular intervals. In some such cases two stars, separated by hardly more than their own diameter, have spectra differing as widely as A and K. The stars of such a pair have doubtless been formed out of the same original mass, and it is quite incredible that practically all the hydrogen has segregated in one component, and practically all the metals in the other.

No: we must look to some difference in the physical conditions of the stars' atmospheres to explain the differences in their spectra, and to differences in some *one* important particular. There is no doubt at present that this condition is the *temperature* of the atmosphere and of the star's surface below it. Practically sufficient evidence of this is found in the colors of the stars. Those which are "early" in the spectral sequence—of Classes O, B and A—are conspicuously white or bluish; stars of Class F are yellowish; and of Class G yellow, like the Sun; those of Class K reddish, and of Class M red—as are also those of Class R, while the N-stars are the reddest in the sky, and the only ones which are found beyond the limits of color mentioned earlier, which correspond approximately to Classes B and M.

It has long been realized that the behavior of the line-absorption in stellar spectra is in general agreement with this hypothesis, on which the B-stars are the hottest, and the M-stars the coldest of those in the main sequence. But the full interpretation of the results has come only very recently, since the processes by which spectra are produced have begun to be understood. The great astrophysical importance of these considerations was pointed out, a year or two ago, by Dr. Saha, Professor at Calcutta, and much of what follows is borrowed from his discussions.

It is now generally accepted that the atoms of the various chemical elements are complicated structures, consisting of a very minute and heavy nucleus, carrying a positive electrical charge, surrounded (at a great distance in comparison with its own size) by electrons, in numbers just sufficient to neutralize the charge on the nucleus (so far as its action on things outside the atom is concerned), and arranged in some sort of successive rings, layers, or shells—each much larger than the last—though even the outermost is perhaps a hundred-millionth of an inch in diameter. These electrons are probably not at rest, but in motion; and there appear to be certain possible *steady states* of motion, in which they can continue to move for long intervals, if not indefinitely. While this happens, the atom neither emits nor absorbs radiation; but if one of the electrons is shifted from one of these steady states to another, radiation takes place. If the electron has less energy in the new position than in the old, the balance is liberated in the form of monochromatic radiation—a train of waves of a definite rate, or frequency, of vibration. If the change is in the other direction, the energy necessary to effect it is absorbed. In either case, the frequency of the radiation is exactly proportional to the amount of energy involved in the change of state on the part of the electron—the factor of proportionality being the "quantum constant" h, which keeps cropping up in all sorts of physical problems, but whose real nature still evades our analysis.

When the energy change is large, the vibrations are very rapid. For example, if one of the innermost electrons, close to the nucleus, is shifted to an outer level, and falls back again, we get radiations of very short wave length—X-rays, in fact. But if one of the outer-

most electrons of all is shifted, the attraction of the nucleus upon it is relatively small, and is modified, too, by that of the other electrons—less work is done, and the corresponding waves are long enough to be called light.

There are many of these possible states, and still more ways in which a shift from one to another may take place. This explains why the spectrum of a given element may contain a great number of lines. Each individual atom at a given instant probably absorbs or emits but one kind of light—a single line—but while one change is going on in one atom, different changes are happening in others, giving different lines, and so the mass of gas as a whole absorbs or emits them all.

In a gas at low temperature—or at least at the lowest temperature at which the element in question can be vaporized—almost all the atoms will have their outer electrons in the state corresponding to the smallest content of energy. When light is passed through this gas the spectral lines corresponding to a shift of the electron from this position to those others which (so to speak) can be directly reached from it, will be absorbed. In a hotter gas, where the atoms collide with each other more violently and increase their stores of internal energy, the light will find some of them with their electrons already in the "higher" positions of greater energy content, and the shifting of these electrons to still higher energy levels will cause the absorption of additional lines. At first there will be relatively few atoms in these other states, and the new lines will be faint. But as the temperature is raised still more, the proportion of atoms which are thus stimulated by collision will rise, and the new lines will increase in intensity, compared with the "ultimate" lines which at first appeared alone.

This reasoning furnishes a beautiful explanation of the manner in which an element such as iron shows relatively few lines when heated just to its boiling point in an electric furnace, more lines when the furnace is heated several hundred degrees hotter, and still more when the atoms are subjected to the much more intense excitation of the electric arc.

But this is not the whole story. If the first and most easily movable electron—of which alone we have been speaking so far—receives a sufficient supply of energy, it may be pulled clear away from the atom, leaving a positively charged, or "ionized" residue. Then the game begins again, a second electron being shifted, stage by stage, and at last removed. During this process the energy-steps are quite different (and, in general, greater) so that we get an entirely new set of spectral lines, usually with more rapid vibrations, and shorter wave-lengths. For some atoms, such as sodium, the second electron is exceedingly hard to get off, and the corresponding radiations are of such short wave-length that they can only be observed with special apparatus. In others, as magnesium, a second electron comes off fairly easily—about twice as hard as the first—giving lines in the visible spectrum and the ultra-violet. These are the characteristic spark lines—or "enhanced" lines, as Sir Norman Lockyer called them, because, even when present in the arc spectrum, they are strengthened in the spark. For magnesium, the third electron is very hard to get away, and the lines corresponding to its removal are difficult to observe. In some other atoms, such as silicon, a third or even a fourth electron may be removed with the production of visible radiation; but, even then, a state is finally reached in which only radiations of too short wave-length to be de-

tected by ordinary means are produced by further stripping of the electrons from the atom. The amount of work which is required to remove an electron from an atom may also be measured by electrical means in a suitably designed vacuum tube, as well as by the spectroscopic processes previously described. Combining the information that we get in the two ways, we know that some elements, like sodium and potassium, are easily ionized, others, like magnesium and zinc, with more difficulty, while the permanent gases, hydrogen, oxygen and nitrogen, are much harder to ionize, and helium hardest of all.

We can now understand what happens in the atmospheres of the stars. In the red stars, where the surface temperature is lowest, the atoms are subjected to a relatively mild stimulus, and the "ultimate," or "furnace" lines of the metals are prominent, with the arc lines of the same substances present, but fainter. For hydrogen, whose ultimate lines lie in the ultra-violet, the only accessible lines belong to a "subordinate series," absorbed only by atoms in which the electrons have already suffered one shift; and at this temperature very few of the hydrogen atoms undergo this internal change, so that the hydrogen lines are weak, and those of the other permanent gases, which are still harder to excite, are absent. As the temperature rises to that of the Sun, the lines of oxygen begin to appear. The arc lines of the metals grow stronger. Some of the metallic atoms lose one electron and are in a position to absorb the enhanced lines. For other elements, such as potassium, which have no enhanced lines in the accessible region, the lines of the neutral atoms gradually fade out, with nothing to replace them. By the time that we reach the F type, the arc lines of the metals are getting fainter, though the enhanced lines

remain, while the hydrogen lines get stronger and stronger. Farther on, in the A type (Sirius) the metallic lines all become weak and just beyond, in Class B9 the lines of helium appear—to strengthen with rising temperature, till at the top, in Class B0, practically all the metallic lines are gone, the hydrogen lines are fainter and we have the helium lines, the enhanced lines of oxygen and nitrogen, and even traces of the enhanced lines of helium. In the atmosphere of such a star—which must be exceedingly hot—the atoms of the metals have almost all lost two, or possibly three, electrons, and very few or none are left in a condition to absorb any lines in the visible spectrum or the nearer ultra-violet. Though the hydrogen atoms are harder to ionize, most of them have lost their one electron, leaving much fewer to produce the line absorption than in the case of the cooler stars of Class A. The oxygen and nitrogen atoms have lost an electron, and even helium—the hardest nut of all to crack—is beginning to yield.

We obtain thus a clear and consistent picture of the processes which underlie the spectral sequence. All that needs further to be said concerns the bands which appear in the spectra of the reddest stars. These are known (in many cases at least) to be produced by compounds. When an electron can be removed from a molecule, containing several atoms held together by those forces which we ordinarily call "chemical affinity," the energy changes are again quite different, and very complex, so that an enormous number of lines are produced, grouped into bands in the spectrum. At the temperature of the M-stars, this happens for the titanium oxide vapor in the atmosphere. But at the higher temperature of Class K the violence of collisions with other atoms is too much for

these molecules: they break apart into their constituent atoms, and are dissociated. The bands disappear, while the lines of titanium are reinforced, the oxygen, meanwhile, going as it were into hiding and requiring a much more violent stimulus to rout it out. But why should the other red stars of the R and N types show a different set of bands, and why should these and the bands characteristic of the M-stars never be found together? Curtiss has given a beautiful explanation. Both sets of bands are due to compounds—one to a metallic oxide—the other to some compound of carbon. It is very likely indeed that these two are chemically incompatible, and incapable of existing together. When the carbon is in excess it takes away the oxygen from the titanium—as it does from iron-ore in the blast furnace. When the oxygen is in excess it combines with all the carbon—getting it into some form in which it does not produce the characteristic bands—and what is left over combines with titanium. So here, at a temperature low enough for the chemical compounds to form, we have a real chemistry of the stars—not merely physics—and our first distinction seems to be the old familiar one between an oxidizing and a reducing atmosphere.

We might now reasonably expect to find that the hottest stars were the brightest, and the cooler stars fainter, but the facts are against us. Among the stars of enormous luminosity of which we have already spoken, those in Orion's belt are of Class B0, at the top of the scale; but Alpha Cygni is of Class A2, Canopus of Class F, and Antares actually of Class M, at the other end of the series. A general study of the relations between the spectral types of the stars and their absolute magnitudes is therefore of importance. The material required for the

elucidation of the real relationship was first provided, a dozen years or so ago, by the investigation of the spectra of faint stars at Harvard; and, as soon as this became available the main features of the situation were pointed out, independently, by Hertzsprung and myself. If we group the stars according to their spectra, we find that those of Class B are always bright. Among the long list studied by Kapteyn, those of Class B2 or "earlier" are almost all brighter than the absolute magnitude zero, that is, more than 100 times more luminous than the Sun. The "later" B's average fainter, and some of the A's come down to less than ten times the Sun's light. In Class F some stars come down to two or three times the Sun's light, yet Canopus, which belongs to this class, is enormously bright. So the list continues, the lower limit of brightness growing steadily fainter, by about two magnitudes from each class to the next, while there is no indication of a change in the upper limit. As far as Class G the stars seem to be distributed all through the range of brightness in which they occur; but in Class K, there appears a division into two groups—brighter and fainter —which in Class M has become very sharp and definite. There are few, if any, stars of this spectral type which are comparable with the Sun in brightness, while there are many whose light is from 20 to 100 times the Sun's, and others with a brightness of from 1/50 to 1/500 that of the Sun.

Combining all these results, we find that practically all the stars fall into one or other of two great divisions. In the first of these the brightness is very much the same, whatever the spectral type, and averages about fifty times that of the Sun: in the other the brightness falls off very rapidly with increasing redness, stars of Class G averag-

ing about as bright as the Sun, and those of Class M a hundred times fainter. Hertzsprung has called these by the happily chosen names of "giant" and "dwarf" stars. The members of either group show a dispersion in brightness of a magnitude or so on either side of the mean—and more in scattering instances—so that the two groups are really separated only in Classes K and M. Even in Class K, a few stragglers occupy an intermediate position, and in Class G the brighter dwarfs and the fainter giants overlap. In Class F the two are much intermingled, and in Classes A and B there is no real distribution. Almost all known stars belong to one or other of these divisions. The few enormously luminous stars, such as Rigel and Antares, may be regarded as scattering giants, much brighter than the average. There are, however, a very few white stars of very low luminosity—the faint companions to Sirius and to Omicron Eridani, and one or two others—which fall definitely outside the scheme, and present an unsolved riddle.

The recognition of the giant and dwarf stars helps to clear up a number of puzzling things—particularly some connected with apparent contradictions in the properties of the "average star" of some given kind. For example, it is certain that a large majority of all the stars in a given region of space (say within twenty parsecs of the Sun) are dwarfs—most of them faint and red. Yet, among the stars brighter than the sixth magnitude, the large majority are giants—as is shown both by direct measures of parallax, and by the study of their proper motions. This is really simple enough. When we make a list of the stars visible to the naked eye, we include only those dwarfs which lie very near us, for, since they are intrinsically faint, the remoter ones are invisible to us.

But the giant stars, being far brighter, can be seen at great distances. We are fishing for them in much wider waters, and naturally we catch more. The most striking instance is afforded by the M stars. Here the dwarfs are so faint that not one of them is visible to the naked eye, though three of the four nearest stars belong to this class, and our naked-eye list is composed exclusively of giant M's, though there are probably more than a hundred dwarf M's which are nearer us than any of these giants. Among the K-stars, only a few of the nearest dwarfs are visible to the unaided eye, while the proportion among the G's is larger. We are therefore dealing with complex mixtures, composed of giants and dwarfs in different proportions for the different spectral types, and the average parallax, proper motion, or what not, of the stars selected in this curious way may be very far indeed from representing the average of the stars of the same type in a given region of space. Indeed, different methods of selection—for example, the rejection of the stars of large proper motion—may greatly change the relative proportion of giants and dwarfs in our mixture, and therefore the calculated characteristics of the "average star." Much care is necessary in interpreting such averages, for the effects of this "involuntary selection" are often far-reaching and unexpected.

One would naturally expect that these great differences in absolute magnitude, among stars of the same spectral class, would be associated with differences in the spectra themselves. From what has already been said, no large differences can be anticipated; but smaller differences exist, and are of much importance. Here again Hertzsprung was the first in the field—pointing out, as long ago as 1905, that certain stars, in whose spectra Miss Maury, at

Harvard, had noticed that the lines were exceptionally narrow and sharp, were much superior in absolute magnitude to the general run of stars.

The greatest advance, however, was made eight or ten years later by Adams and Kohlschütter. By this time reliable information was at hand regarding the absolute magnitudes of a large number of stars. Photographing the spectra of a selected list of these with the same instrument, and comparing the plates, the Mount Wilson investigators were able to detect many small differences which had escaped earlier notice. Certain lines were stronger in the giant stars, and others in the dwarfs. What is more, the relative intensities of certain pairs of such lines (one of each kind) was found to vary in a regular manner with the absolute magnitude. When the law of this variation had once been determined, with the aid of a dozen or so stars of a given spectral class for which the parallaxes and absolute magnitudes were known, it became possible to take the spectrum of any other star of this class, examine the lines in question, and find from their relative intensity what is the star's absolute magnitude—in other words, how bright it really is. Knowing this, and the star's apparent brightness, we can immediately calculate its distance and parallax.

In this way the way opened for the determination of "spectroscopic parallaxes." The method is surprisingly accurate—the average error of a determination of parallax being about twenty per cent.—and very rapid in practice, since it takes much less time to get the two or three spectrograms which are sufficient for the purpose than to take and measure the dozen or fifteen plates which are required for a determination by the direct or "trigonometric" method. Moreover, since the probable error is

a fixed percentage of the parallax, the spectroscopic method is much superior to the other in accuracy for faint stars at great distances, where the parallax is smaller than the errors of observation by the direct method. For the nearest stars, on the other hand, the trigonometric method is the more precise. The most serious limitation of the new method is that, as developed at present, it is available only for stars of spectrum more advanced than A8. Some recent work by Lindblad at Mount Wilson, however, indicates that, by the use of other spectral lines and somewhat different methods the A-stars may be included. As, thanks to Kapteyn, we already know a great deal about the B-stars, this will fill the last serious gap in our information.

Two thousand stellar parallaxes have been determined at Mount Wilson by this method during the past five years, and the work goes on actively. Shapley at Harvard is just beginning investigations of the same sort, using the unique collection of photographs accumulated by Pickering's untiring energy. It is not too much to hope that, within a very few years more, we shall know the distances of all the stars of Class F and redder, which are visible to the naked eye, and perhaps of many fainter ones. If Lindblad's methods fulfil their promise, and Kapteyn's investigations on the B-stars continue, we may arrive before long at a very good idea of the distances and distribution in space of all the stars which can be seen without a telescope.

The results so far obtained confirm completely the existence of the giant and dwarf stars, and their characteristic properties, and give us information about the very brightest stars that we could hardly have hoped to obtain otherwise. In all the spectral classes there are some stars

a thousand times as bright as the Sun, or even brighter. These all show the narrow sharp lines noticed by Miss Maury, and are often called c-stars, following her notation. The average parallax of these stars can be obtained with considerable accuracy from their proper motions, taking into account the apparent drift produced by the Sun's motion in space, so that the spectroscopic parallaxes have here another firm point of support. Some of these c-stars are variable in a highly distinctive fashion, known as Cepheid variation, from the star Delta Cephei. The changes in brightness are continuous, with a regular period of a few days or weeks, and are accompanied by changes in color and spectrum—the star being redder, and further advanced in spectral type, at minimum. Whenever we find a star that varies in this fashion we can be morally certain that it is several hundred times brighter than the Sun.

Now a large number of faint stars which vary in brightness in exactly this way occur in the Magellanic Clouds—those remarkable patches, like outlying fragments of the Milky Way, which lie near the south pole of the heavens. They must be all at substantially the same distance from us. Miss Leavitt, at Harvard, found some years ago that there is an extraordinarily exact connection between the average brightness of these stars and their periods—those of longer period being the brighter. It is extremely probable that this relation holds true also among the nearer Cepheid variables which have already been mentioned. If this is so we can determine the absolute magnitude of such a variable, and hence its distance, from a mere knowledge of its period. Following this line, Shapley has determined the distances of more than a hundred isolated Cepheid variables, some of which

are as much as five thousand times as bright as the Sun. Their distances range from sixty parsecs (for the Pole-star, which shows a slight variation of this type) to more than five thousand parsecs in a few instances. But these are not the greatest distances that can be measured by this very powerful photometric method. Shapley derives a distance for the Smaller Magellanic Cloud, of 19,000 parsecs. More recently Seares, working on similar principles, but utilizing our knowledge of the absolute magnitudes of blue stars (whose color indicates that they are of Class B) has estimated that the star clouds in the Milky Way itself lie at distances ranging from 6,000 to 15,000 parsecs.

Even these distances are surpassed by those of the great globular clusters of stars, which have been the object of one of Shapley's most striking investigations. From a study of the Cepheid variables which appear in these clusters, and by a most ingenious combination of other data, he has succeeded in obtaining reliable estimates of the distances of all the known objects of this type, 69 in all. These clusters are extraordinarily similar in size and constitution. The denser central portion of any one is some 10 parsecs in diameter, while the outlying stragglers extend to a distance of more than 50 parsecs in all directions. Within this region there are at least 40,000 stars brighter than the Sun, and probably a still greater number of fainter ones. The brightest stars, which are almost all red giants, average about 1,000 times the Sun's luminosity.

The nearest of these gigantic systems—which is visible to the naked eye in southern latitudes as a hazy star, Omega Centauri—is 6,500 parsecs away, while the remotest of them is at the enormous distance of 65,000 par-

secs, or more than 200,000 light years. They are scattered somewhat irregularly through a somewhat oval region, symmetrical about the plane of the Milky Way, and almost 100,000 parsecs in maximum extent. The centre of this region is by no means near the sun—as is obvious from the fact that most of the globular clusters are found in one hemisphere of the heavens, and hardly any in the other—but is distant about 13,000 parsecs, in the direction of the constellation Sagittarius. It appears, therefore, that we are very far indeed from being at the centre of the universe. Astronomers, like others, used to think so, and were confirmed in their belief by the fact that the number of stars visible to the naked eye, and even in a small telescope, is substantially the same in all regions near the Milky Way (though smaller, of course, near its poles). This means that such stars are distributed around us almost uniformly in all directions in the galactic plane. But we now believe, with good reason, that the cause of this is that practically all the stars visible with a small telescope lie within relatively small distances—two or three thousand parsecs—which do not reach out to the boundaries of the universe of stars. It is only when we consider very faint stars, which are much more remote, that the real eccentricity of distribution becomes apparent. In other terms, we used to suppose that our soundings of the depths of stellar space gave us the same results in all directions: but what they really recorded was "no bottom." It is only with the longer measuring line afforded by modern photometric methods of determining distance that we strike bottom at last. This expansion of our conceptions of the extent and magnitude of the stellar universe is one of the most remarkable of all the advances of modern Sidereal Astronomy.

II

THE SIZES AND MASSES OF THE STARS

WE described yesterday the close relations which exist between the spectral types, colors, and temperatures of the stars, but we said little or nothing about the actual temperatures of their surfaces. At the beginning of the twentieth century, there was really very little that could be said; but since then our knowledge has increased rapidly.

The laws of radiation from heated bodies are qualitatively familiar to everyone—especially in these days when electric heating is a commonplace. If we heat a body to a moderate temperature, it will radiate energy, which the hand held near by can appreciate as heat—though the eye cannot see it, because it is carried by waves too long, and too slowly vibrating, to affect this organ of special sense. Raise the temperature higher, and the heat radiation increases, and light begins to appear, at first dull and red, then brighter. As the temperature still increases, the light becomes much brighter and changes from red to yellow, and at last to white—till finally our experiment (as in the case of an "overvolted" lamp) is brought to an end by the melting or volatilization of the heated body.

The accepted formulation of these laws in mathematical language is due to Planck. His formula—tested both by observation and by thermodynamic reasoning—is rather complicated to discuss here, but its principal consequences, so far as they affect astronomical phenomena, may be

stated in simple form. They apply primarily to the behavior of a perfect radiator (or "black body" as it is often called, because such a body would be also a perfect absorber of incident radiation, and hence look perfectly black), and, as might be expected in any physical formula, the temperature which appears in it is measured, not from the arbitrarily chosen zero of our thermometers, but from the absolute zero (—273° centigrade, or —460° Fahrenheit) at which a body would contain no heat energy.

Considering first the total radiation of energy, from a unit area, and in a unit time, it is found that this is proportional to the fourth power of the absolute temperature. At 1000° absolute (on the Centigrade scale) the emission is 1.37 calories per square centimeter per second, according to the excellent measures of Coblentz, which should be accurate within a fraction of one per cent. Since the Sun radiates 1490 calories per second from every square centimeter of its surface, a simple calculation shows that, if it were a perfect radiator, its temperature would be 5740° absolute. An imperfect radiator would emit less heat at this temperature, and have to be heated hotter in order to give out the same amount. Hence this figure may be regarded as a lower limit for the actual surface temperature of the Sun.

Taking next the amount of light radiated per unit of surface—that is, the surface-brightness of the body—it appears that the relation between this and the temperature is most simply defined by expressing the surface brightness in stellar magnitudes. We may then say that the changes in surface brightness are proportional to those in the reciprocal of the temperature, so that we have an equation of the form $J = A + B/T$, where J is

the surface brightness, T the temperature, and A and B are constants. This equation is not absolutely exact, but the error only becomes important at temperatures exceeding 10,000°, when the formula gives values which are a little too low. Applying this method to the Sun, using Abbot's measures of the energy which it sends us in different wave-lengths, we find a temperature of 6100°. This also should be a little too low if the Sun is not a perfect radiator. The discordance between this and the previous value shows that such is indeed the case.

For light of different wave-lengths the coefficient B in the above equation varies inversely as the wave-length, indicating that the change of violet light (and hence of photographic magnitude) with the temperature is more rapid than that of visual magnitude. It follows that the variation of the color index with the temperature can also be represented by an equation of the same form, but with different values of the numerical coefficients A and B, the latter being about one-quarter as great as in the case of the visual surface brightness. Applying this method again to the Sun (still using Abbot's data), we get a decidedly lower temperature, about 5200°. The discrepancy can be explained by the fact that the violet part of the solar spectrum is full of dark lines, while there are not nearly so many in the green. If we could get rid of these, the violet rays would be considerably more strengthened than the green, the Sun's light would appear blue, and the "color-temperature" would come out higher. All things considered, we may adopt 6000° as the surface temperature of the Sun, with reasonable assurance that we are within a few per cent. of the truth. We must remember, however, that this is only a sort of average temperature of the various layers in the sun's

atmosphere. The deeper layers are undoubtedly hotter than those nearer the surface, and it is therefore not surprising that at the centre of the Sun's disk, where we look straight down into its atmosphere, and probably see deeper into it, the light is brighter and bluer, corresponding to a temperature about 400° higher than calculated for the integrated light of the whole Sun.

When we attempt to estimate the temperatures of the stars, our first reliance must be upon the method depending upon the color of their light—for, without a knowledge of their diameters, we cannot directly determine the surface brightness, nor the emission of energy per square centimeter. Extensive studies of the distribution of energy in stellar spectra—which is a more precise specification of the color of their light—have been made by Wilsing and Scheiner at Potsdam, and by other observers. Their results are in good general agreement, and indicate that the surface temperature of the M-stars is about 3000° or a little higher; of the K-stars 4000°, of the G's 5400°—agreeing substantially with the color-temperature of the Sun—; for Class F about 7000°, and Class A, 11,000°. The typical stars of Class B must be exceedingly hot, but it is very difficult to determine their temperatures, as a small error in the observed colors makes in this case a large one in the temperature. From all the evidence, it may be estimated that in some cases their temperature reaches 20,000°, or perhaps more. The observed color indices of the stars agree excellently with those which would be exhibited by perfect radiators at these temperatures—which is not surprising, because they depend on observation, in another manner, of the very characteristics by means of which the temperatures were calculated.

Another, and more important, check upon the estimates of temperature can be found in the *luminous efficiency* of the stars—that is, the ratio of their emission of light to their whole out-put of radiant energy, that is, of heat. It is only in recent years that apparatus delicate enough to measure the heat of the stars has been devised. The most successful observations have been those of Coblentz at the Lick Observatory in 1914. With an exceedingly delicate thermopile, placed at the focus of the three-foot Crossley Reflector, and connected with a galvanometer of extreme sensibility, he obtained not merely indications, but measures, of the heat radiation from white stars as faint as the third magnitude and red stars at the limit of visibility to the unaided eye. More than eighty stars in all were measured, distributed among all the principal spectral classes. From these results it is found that stars of the same spectral class all give about the same amount of heat in proportion to their light, but from one class to another the proportion is very different. The whiter stars, of classes B, A, and F, give the most light in proportion to their heat, and are very similar to one another. But, for the same visual radiation, stars of Class G send out about 70 per cent. more heat, those of Class K more than twice as much, and the M-stars six times as much. The extreme case is Alpha Herculis, which has a spectrum of very pronounced M type, and gives out more than thirty times the heat, in proportion to its light, which an A-star does—or only three per cent. as much light, in proportion to the total heat.

Now it is well known to all illuminating engineers that the luminous efficiency of artificial sources of light increases very rapidly with the temperature—which is why

tungsten lamps, which work at a higher temperature, give us several times as much light for our money as the old carbon filaments. The same principle, applied to the stars, confirms the low temperatures of Classes K and M. But the white stars present an apparent difficulty. As a black body is raised to very high temperatures—well above that of the Sun—most of the energy which it radiates passes into the ultra-violet, where it does not affect our eyes. Hence, theoretically, the luminous efficiency should be a maximum for a temperature not far from the Sun's, and fall off again for the hotter stars. The solution is found in the fact that the observations were made with light which had come through the earth's atmosphere, and been reflected from a silvered mirror. Our atmosphere is entirely opaque to all ultra-violet light of wave length less than 3000 Angstrom units (¾ that of the visible violet) and silver, though an admirable reflector for all visible rays, hardly reflects at all in the ultra-violet below 3000 and 3300. Hence practically no radiation of shorter wage length than this and very little beyond 3500, reaches the thermopile, and the powerful ultra-violet radiation of the white stars fails to record itself. When allowance is made for this, and also for the absorption of the long infra-red wave in the atmosphere, we get the ratio between the amount of the light of a distant hot body and that portion of its heat which could have reached the measuring instrument. Calculating this, with the temperatures already assigned to the various spectral classes, we obtain results which are in excellent agreement with Coblentz's observations throughout their whole range. For the coolest stars, this affords a better method of measuring temperature than the color index, which may be considerably

affected by the presence of heavy dark bands in the green, yellow and red. In this way it may be calculated that the surface temperature of the one N-star which was observed by Coblentz is about 2500°, and that of Alpha Herculis but a little more than 2000°.

With our estimates of temperature thus confirmed, we may proceed to calculate the relative surface brightness of the stars. Taking the Sun as typical of Class G, we find that a star of the same size, but of Class B0, should be perhaps thirty times as bright, one of Class A0 ten or twelve times as bright, one of Class F0 about three times. The surface brightness of a K0 star should be about 1/7 of the Sun's and that of an M-star about 1/50. All these values, of course, are averages, and individual stars may sometimes run somewhat above or below them, especially at the extremes of the spectral scale. As an extreme example (to judge by its luminous efficiency), Alpha Herculis may have a surface brightness as low as 1/500 that of the Sun.

We are now in a position to form an idea of the real sizes of the stars. Take, for example, Sirius, which is 25 times as bright as the Sun, and of Class A. Taking its surface brightness as ten times the Sun's, its superficial area comes out 2½ times the Sun's and its diameter 60 per cent. greater than that of the Sun, or 1,200,000 miles. Procyon, of Class F5, six times as bright as the Sun, and probably twice as bright per unit of surface, must be nearly equal to Sirius in diameter. Alpha Centauri, which is almost exactly similar to the Sun in brightness as well as in spectrum, must be also very nearly of the Sun's diameter, 866,000 miles. Passing to redder stars, we may take 61 Cygni. The components of this well-known double star, one of our closest neighbors in

space, give out respectively 1/20 and 1/40 of the Sun's light. Their spectra are K7 and K8, and their probable surface brightness something like 1/30 and 1/35 of the Sun's, which would make their diameters about 1,100,000 and 800,000 miles. A typical dwarf M-star may be taken as giving 1/150 the Sun's light, which, with a surface brightness of 1/50, would make its diameter 60 per cent. of the Sun's, or 500,000 miles. Finally, we may take "Barnard's star," the nearest after the system of Alpha Centauri, which is only 1/2700 as bright as the Sun. Its spectrum is of an advanced M type, and we may perhaps estimate its surface brightness as 1/300 of the Sun's, which would make its diameter some 300,000 miles. All these are dwarf stars, and have been chosen as fairly typical examples. The conclusion that such stars were usually from a couple of million to half million miles in diameter would be confirmed by further evidence.

For the giant stars, however, things must be very different. Take first the stars in Orion's belt, which average 4000 times as bright as the Sun. They are of Class B0, and their surface brightness may be estimated as thirty times the Sun's, which would correspond to a diameter of just about ten million miles. For Rigel, of Class B8, and 13,000 times the Sun's luminosity, the surface brightness may be estimated as 12 times the Sun's and the diameter about 30 million miles. Arcturus, a good example of the ordinary run of the redder giants, is about 90 times as bright as the Sun. Taking its surface brightness as 1/7 of the Sun's, since it is of Class K0, we find a diameter of 25 million miles. At the extreme of size come those stars which are at the same time extremely bright and very red. Betelgeuse, in Orion, is of Class M, and, according to the best meas-

ures of parallax, appears to be 900 times as bright as the Sun. With a surface brightness 1/50 that of the latter, the estimated diameter reaches the enormous value of 210 million miles. Antares, of similar spectral type, and 3000 times brighter than the Sun, by the same token comes out 390 million miles in diameter. These last values are, however, exceptional. We know of no other stars which bid fair to rival them, except perhaps Alpha Herculis, which according to Adams, is 250 times as bright as the Sun. With our previous estimate of surface brightness, its diameter would be 300 million miles —but this particular estimate is exceedingly uncertain.

These conclusions are so remarkable that they require to be tested in every available way. Until within a year or two, there was only one direct way of finding the diameter of a star, and this could be used in only a few instances. There are a good many stars which show regular, periodic, variations in their light, which give every evidence of arising from eclipse—the star being really double, with components so close together that they hide one another, partially or completely, during every revolution. If we have good measures of the brightness of such a star throughout its changes, we can find the relative diameters and brightness of the two stars—can indeed make a complete map or model of the system, except that we do not know the actual dimensions, that is, on what scale our model is drawn. But when, in addition, the star has been observed with the spectroscope, and both components are bright enough to be recorded, we may find the velocity of their orbital motion, and so put a scale of miles upon our model, and obtain the actual diameters of the stars. About a dozen such systems have so far been investigated. All these stars but one are bigger than the

Sun, and most of them are between twice and four times its diameter. As they are of Classes B and A, this confirms our estimates, when it is remembered that the stars in Orion's belt are unusually luminous, even for B-stars, and therefore probably larger than the average. The one star which is smaller than the Sun (about two-thirds its diameter) is a dwarf (W Ursae Majoris) of spectrum F8, and again supporting our conclusion.

But until last year, there was no direct confirmation of the great diameters computed for the redder giant stars. This has been furnished by one of the most notable advances in observational astronomy in the present generation—the direct measurement of the angular diameters of stars by Michelson and his associates at Mount Wilson. The great difficulty in the way of such measurements lies partly in the enormous distances of the stars, but mainly in the properties of the waves of light themselves. Though these travel in straight lines in empty space, yet, when they pass a sharp obstacle they have a slight tendency to bend inwards into the shadow: just as sea-waves passing the end of a long breakwater would spread sidewise into the calm space behind it. This spreading, called diffraction, is greater the narrower the aperture through which the light is admitted. With a very narrow slit, it becomes conspicuous, as may be seen by looking at a distant object through the gap below two lead-pencils held close in front of the eye, and pressed so close that light barely gets between them. With larger apertures, it is evident only under considerable magnifying power. The "diffraction pattern" produced by the circular aperture of a telescope, when sufficiently magnified, appears as a circular disk of light, brightest at the centre, and fading off gradually toward the edge, surrounded by a succession of concentric

luminous rings, separated by dark spaces and each fainter than the last. Even if the object at which the telescope was directed were a mathematical point, its image would still show this spurious disk, and, for a larger object, the image of each point is similarly expanded, so that the whole effect is blurred. No perfection of workmanship can eliminate this difficulty, which is inherent in the very nature of light; but it may be reduced by using a telescope of large size, for, the larger the aperture of the telescope, the smaller is the spurious image. For a one-inch aperture, it is 4″.5 in diameter, but for a ten-inch it is only one-tenth as large. For the 40-inch Yerkes telescope, the diffraction disk measures only 0″.11, and for the 100-inch at Mount Wilson it should be slightly less than 0″.05. A large telescope, therefore, is inherently superior to a small one in its capacity for revealing fine detail, such as close double stars. When, with a small instrument, the spurious disks would overlap and conceal all trace of duplicity, a great one shows the components sharply separated—provided, at least, that the air is steady. On nights when the air is not optically homogeneous—which, alas, are in the large majority at most places—there is a continuous "boiling" or disturbance of the image, which seriously increases the blurring, and too often renders all refined observations impossible.

From the known distances of the stars, and the real diameters already estimated, it is easy to show that, even in the most favorable instances, the angular diameter of a star will not exceed 0″.05, and that cases in which it is greater than 0″.01 are rare. The real disks of the stars, therefore, must be smaller than the spurious disks produced by diffraction in even the greatest existing telescopes, and all that we can hope to see, even under ideal atmos-

pheric conditions, is the latter. This makes the problem seem hopeless—barring the construction of huge telescopes, far larger than any now in existence, and fabulously costly. But Michelson—a life-long student of light waves and their uses—has flanked the obstacle by the apparently mad course of blocking up most of the opening of the telescope, and letting the light come in only through two apertures on opposite sides of the centre. The light from one of these apertures alone would produce a diffraction disk (or a figure of some other shape, if the opening were not circular) of much larger size than that given by the full aperture of the telescope. But, when the images from the two apertures are superposed, a new phenomenon appears. The large fuzzy spurious disk is crossed by fine, parallel, equidistant, dark bands, or "fringes" of exquisitely clear definition. These fringes are produced by the "interference" of the light waves of the pencils proceeding from the two apertures, very much as the spurious disk is produced by the interference of waves from all parts of the full aperture: but the distance between the fringes is inversely proportional to the separation of the two apertures, and when these are set at opposite sides of the full opening, the fringe width is eighty per cent. of the diameter of the spurious disk due to the unobstructed aperture.

Suppose now that we have a very close double star—so close that, even with the full aperture, the spurious images of the components are confused. If we pass to the two apertures, we shall have larger spurious disks than ever, and more completely superposed, but each will be crossed by its own system of fringes, and the central fringe of each system will pass through the geometrical image of the corresponding star. If these geometrical images are separated by only half the distance of the fringes, the

bright fringes produced by one star's light will just occupy the dark spaces between the fringes given by the other, and the system of fringes will disappear. This happens when the components of the double star are of the same magnitude. If one of the stars is slightly brighter than the other, the "visibility" of the fringes diminishes, and they assume a washed-out appearance, without vanishing entirely. In either case, the existence of a double star can be detected, although the components are separated by less than half the distance at which their spurious images would begin to be confused, if observed with the full aperture. By making the distance between the apertures adjustable, and arranging them so that the line joining them can be turned to cross the main aperture in any direction, it is possible to measure the distance of a close double star (changing the distance of the slits until the fringes vanish). In this way measures were made upon the bright star Capella, which has long been known, from spectroscopic measures, to be a very close binary, with a period of 104 days. No telescope has ever resolved the components, but, with the interferometer, immediate evidence of the duplicity was secured. What is more, and better, it was found that very precise measures could be made, following an ingenious method devised by Anderson, although the distance was as small as 0″.04. Indeed, the precision of the results appears to be fully twenty times greater (as regards the distance measures, at least) than can be secured by any method of observation previously known. Best of all, it was found that atmospheric disturbances—contrary to expectation—had much less influence upon the definition of the fringes than upon that of the star-image as a whole. This is conspicuous even to the novice who has the privilege of observing with the instrument. While

the star images dance about, the fringes remain almost fixed, and present a singular appearance of objective reality.

So far we have spoken as if the single stars of the pair were mere luminous points. If the interferometer is turned upon an object of finite angular diameter, and the distance between the apertures increased, the fringes gradually fade away and finally disappear. How this happens may be seen by imagining the star-disk to be divided into two semi-circles by a line perpendicular to the position angle of the interferometer apertures. Each half-disk is an independent source of light, and will give its own system of fringes. As the apertures are separated and the fringes become narrower the two sets will finally become superposed, giving a uniform illumination. When this happens, the reading of our instrument (assuming that we interpret our observations as in the case of a double star) will give the distance between the centre of the luminous area of one semi-circle and that of the other. It is easy to see that this distance will be rather less than half the diameter of the whole circle—in fact it is about forty-one per cent. of it. Our readings may, therefore, be calibrated to give at once the diameter of the star-disk.

The resolving power of the interferometer is less in this case than for a double star, and a greater base line than even the diameter of the 100-inch telescope was desirable. An auxiliary apparatus was therefore adopted, consisting of a rigid steel beam twenty feet long, fixed across the outer end of the telescope, carrying two mirrors inclined at 45° to the axis of the beam, and capable to being set at any desired distance apart, up to the full light of the beam. These reflect the star's light inward along the beam to another pair of diagonal mirrors, which send it into the telescope. With this device the effective distance between

the "apertures" is that between the centres of the outer mirrors, and the power of the instrument is more than doubled. Working with this, Michelson and Pease, in December 1920, found that the fringes disappeared in the light of Betelgeuse (Alpha Orionis) when the mirrors were ten feet apart, while in the case of other stars the fringes were clearly visible with a much wider separation—proving that the cause of their disappearance was not any defect of the instrument, or atmospheric disturbance, but a measurable diameter of Betelgeuse. Calculation showed the diameter to be 0″.047—a small quantity according to ordinary standards, but corresponding to enormous dimensions in miles, for the parallax of this star (according to the average of the results of several good observers) is only 0″.020. The diameter of the star is therefore 2.3 times the distance which separates the Earth and the Sun, or 215 millions of miles. In the following summer Pease extended this success by measuring the diameter of Arcturus, 0″.022, and of Antares, 0″.040. The parallaxes of these stars, ″.097 and ″.009, are known with greater percentage accuracy than that of Betelgeuse, and the computed diameters are correspondingly more trustworthy. That of Arcturus comes out 21 million miles and of Antares 400 million—considerably larger than the orbit of Mars.

A few other stars, notably Aldebaran, probably have angular diameters large enough to measure in the same fashion. To measure smaller stars, it might appear neccessary to build a still larger interferometer. But this has been avoided by Michelson's ingenuity. By a very simple device, he is able to produce, in the same field of view as the fringes already described, another set, formed by light from the same star and adjustable to any desired degree of visibility. In this way the visibility of the main fringes

may be accurately measured, and this tells us how far towards disappearance they are when the mirrors are set at a given distance—say twenty feet—and hence at what distance the mirrors would have to be set to give complete disappearance, even though this may be far beyond the mechanical limits of the apparatus. This will multiply the power of the instrument three or four fold, if not more, and may make it possible to measure a few of the white stars, such as Sirius. This solution of a century-old problem, in its brilliancy, simplicity, and rapid advancement, deserves to rank with the most noteworthy triumphs of physical investigation, and would doubtless have brought a swarm of honours upon the head of its inventor if he had not already received almost all that the scientific world has to give. With its aid one may reasonably expect to have, within a few years more, a sufficient number of measures of stellar diameters to tell us whether the theoretical predictions will be as good in other cases as they are in these three. It may be mentioned in passing that the estimates of temperature for the various classes, upon which the calculations given above depend, are taken from some notes of mine prepared about three years before Michelson's apparatus was projected. Indeed, before Michelson's results appeared, Wilsing in Germany, Eddington in England, and Shapley and I in this country, had all published predictions concerning the diameters of Betelgeuse, none of which differed by more than about thirty per cent. from the value later observed. Unless further observations with the interferometer reveal larger discordances than have so far appeared, it should soon be possible to obtain thoroughly reliable values for the surface brightness of the stars of the various spectral classes, and then to calculate the diameters in miles of all stars of known

parallax, and the diameters in seconds of arc of all the stars in the sky—except perhaps for a few abnormal objects.

We can be sure then, that, while the dwarf stars are all much alike in size, the giants are huge affairs—so huge, in some cases, that their size would be almost incredible if the evidence were not conclusive. The bulk of these bodies is amazing. Arcturus—not a large star as giants go—must have about fourteen thousand times the cubic content of the Sun, while Antares exceeds the Sun in volume a hundred million fold! One naturally asks, Is the quantity of matter in such a star—its mass—proportionally as great as its bulk? To answer this question in the case of any star, we must have recourse to gravitational theory. We can measure the mass of an astronomical body only by the effects of its attraction upon the motion of some neighboring body—and this limits our knowledge of stellar masses to double stars. Fortunately these are so numerous and so well distributed among the various spectral classes, both giant and dwarf, that we can be reasonably sure that they present a fair sample of the general run. When the stars of a binary pair have completed a large part of a revolution (or more) while under observation, their orbit can be computed, and then, if the parallax is known, the mass of the pair is easily found. Even if the motion is so slow that only a few degrees of the orbit have been described in a century, it is still possible to get valuable information by a statistical process, which gives values which may be considerably too large or too small for any individual star, but leads to an average which will be nearly correct for groups of twenty stars or more.

Combining these methods, several hundred systems be-

come available. Recent work by Jackson at Greenwich shows that the average mass of a pair of dwarf stars is very close to twice that of the Sun, so that the individual components are strictly comparable with the Sun in mass. There is evidence, however, that the average mass differs for the various spectral classes. Ludendorff showed, a decade or more ago, from the data regarding spectroscopic binary systems, that the average mass of a pair of B-stars was about three times that of those of Class A and more than ten times that of the Sun. This conclusion was confirmed and extended by some work of mine a little later. Seares, who has recently gone over the same ground with more extensive data, confirmed these results and finds that the mean mass of a pair of stars of Class B0 is eighteen times that of the Sun, 10 for A0, 4.4 for F0, and 1.7 for G0, then diminishing slowly to 1.4 for M. For classes B and A these values apply to the giants and dwarfs together, for the later classes to the dwarf stars. For the redder giants the average mass of a pair is about 9 times the Sun's, according to a determination of mine, which I hope to revise in a year or two, but which is probably fairly reliable. These mean masses show that the stars are far more similar to one another in mass than in any other characteristic. This conclusion is confirmed by the individual values. If we confine ourselves to cases in which the errors of observation are not so large as to vitiate the results, we find no case of a mass exceeding twenty times that of the Sun, and none less than one-seventh of the Sun's. There appears to be a definite correlation between mass and brightness. The large masses all belong to B-stars of great luminosity—the smallest one to a tiny M-star, about 1/3000 as bright as the Sun. The average values for the giant and dwarf stars point in the same direction.

We are now in a position to evaluate another very important physical characteristic of the stars—their mean densities. Among the dwarfs, we have seen that the average mass of an individual star diminishes slowly from twice that of the Sun for Class F to half the Sun's for Class M, and that the radii ran from sixty per cent. greater than the Sun to forty per cent. less than the Sun's. This shows that the densities of all these stars are of the same order of magnitude. If we set the average density of a dwarf F-star as half of the Sun's, of a G-star as equal to the Sun's and of an M-star as twice as great, we shall be in good agreement with our data. For a very faint star like that mentioned above, we may estimate a diameter one-third and a mass one-seventh of the Sun's, giving a density four times that of the Sun, or about equal to the density of the Earth. Our knowledge of the surface brightness of these dwarf stars is not accurate enough to justify much reliance on the differences between these figures, but the general conclusion that the dwarf stars are comparable in density with the Sun, and most of them denser than water, appears certain. For the stars of Class A, we are on firmer ground, for most of the eclipsing variables belong here. In every such system, where we know the period and the relative sizes of the stars and orbit, a fortunate mathematical relation permits us to compute the density without having to know the dimensions or distance. For 69 stars, of spectrum between B5 and A5, the average density is 1/5 that of the Sun. They are decidedly similar to one another—⅞ of them having densities between four times the mean and ¼ of it. The few eclipsing variables of early B type indicate a much smaller mean density—about 1/40 that of the Sun.

Passing to the giant stars, we may take as an example of

Class G, the brighter component of Capella. This star is 70 times as bright as the Sun and has 4.6 times its mass. Assuming that its surface brightness equals the Sun's we obtain a volume 600 as great, and a mean density 1/130 of the Sun's, or between eight and nine times that of air under standard conditions. For Arcturus, with a volume 14,000 times the Sun's 10 times the Sun's mass would be a liberal estimate. The corresponding density is 4/5 that of air. Finally for Antares, with its enormous bulk, even if we assign the exaggerated and improbable mass of 100 times that of the Sun, the density comes out one-millionth of the Sun's, or 1/900 that of ordinary air. The actual density is probably somewhat less, but more than 1/5 as great.

These values are so extraordinarily small that confirmation is desirable—and the eclipsing variables provide it. Of those of "later" spectral type, the majority are dense stars, one or two nearly equaling the Sun, and one having almost twice the Sun's density. But in a few cases the density is very much less. In one case it is about 1/100 that of the Sun, in two others 1/600 and 1/2000, and finally in one remarkable star, W Crucis, there is clear evidence of a density only 1/500,000 of the Sun's, or about 1/400 that of air. With this independent evidence, the reality of these low densities is placed beyond doubt. But when we realize that the mean density of Antares or W Crucis is less than that of what was once considered a fairly good vacuum, before air-pumps reached their present degree of perfection, we can but ask, Can a body of such low density possibly be opaque, and shine as if it were solid? We must not forget, however, that even a perfectly pure gas inevitably scatters some light away from a beam which passes through it, owing to the action of the

very molecules themselves. The atmosphere above our heads—equivalent to five miles of air of standard density—scatters about one-eighth of the incident light, even when perfectly free from dust. After passing through fifty miles of air, only thirty per cent. of the original beam would remain; at a hundred miles, nine per cent.; after two hundred, less than one per cent.; and at the end of six hundred miles, not one part in a million. Such a thickness of perfectly clear air would be substantially opaque—the Sun itself would be barely visible through it. Now an easy calculation shows that a ray of light which passed centrally through Antares (taking the star as of uniform density, simply for the purpose of this calculation) would in our lowest estimate have passed through a quantity of gas equivalent to 800,000 miles of air—or more than a thousand times enough to secure substantial opacity. We need, therefore, trouble ourselves no more upon this score.

The very low densities of the giant stars permit an explanation of the spectroscopic differences upon which Adams has based his determination of the distances of the stars. The temperature of the atmosphere of a star of a given spectral class is probably much the same, whatever its absolute magnitude, but the density of this atmosphere, like that of the whole mass, must be very much greater in a dwarf star than in a giant. What spectroscopic differences might arise from this can be seen upon Saha's theory.

The relative strength of the ordinary lines and enhanced lines of any element depends upon the relative number of the atoms of this element, in the star's atmosphere, which are in the neutral and ionized states. Now an atom becomes ionized when its internal energy becomes so great as to lead to the ejection of an electron, and the rate at which this happens will be practically independent

of the density of the gas, though it will increase rapidly with the temperature. The rate of recombination of the ions thus produced with the free electrons, to reconstitute neutral atoms, will, however, be greatly influenced by the density. It is obvious that the farther the atoms and electrons are apart, the longer, on the average, they will have to travel before meeting one another. Hence, though the atoms will not *enter* into the ionized state oftener in the gas of low density, they will *remain* in it longer, and the percentage of all the atoms which are ionized will increase as the density diminishes. This will favor the absorption of the enhanced lines at the expense of the lines of the neutral atom. Now the lines which Adams has picked out, on the basis of his wide experience, as characteristic of giant stars, are almost without exception enhanced lines of a pronounced type. The general character of the physical process which lies behind his beautiful discovery appears therefore to be understood. Many details remain obscure —for example, the prominence in the giant stars of the hydrogen lines, which belong to the neutral atom. But in so new a field of work, the presence of such unexplained phenomena is only a stimulus to further research, and a comforting reminder that there are still "new worlds to conquer."

III

THE CONSTITUTION AND EVOLUTION OF THE STARS

THE preceding lectures have, I hope, presented to you a picture—drawn rather in outline, owing to the limitations of time—of the stars as they are known at present, their dimensions, masses, densities, surface brightness, and the like. It remains to speak of what has been done to correlate these facts into a theory of the constitution of the stars, and their probable evolution and age.

What makes this problem tractable, in spite of the limitations imposed by the remoteness of the stars in space, and our ephemeral duration in time, is that we have to deal only with the simpler and more general properties of matter. The vast variety of the forms of rock and mountain depends upon the solidity of their materials; the still greater diversity of the forms of organic life is based on the presence of chemical compounds of great complexity—and neither of these conditions can exist at all in bodies as hot as even the coolest stars. In the stars all matter must be gaseous, and the laws of gases are among the simplest known to physics. Add to them the still more general laws that govern gravitation, radiation, and the structure of atoms, and we have the controlling factors in the evolution of the stars.

Considering a star, then, as a mass of gas, isolated in space, we notice first that it must be in internal equilibrium

under its own gravitation. The weight of the overlying layers produces a pressure increasing steadily from the surface to the centre, which must at any point be balanced by the expansive tendency of the gas, arising from its high temperature. The temperature, too, is greatest at the centre, and decreases towards the surface. Hence, heat must flow continually through the star's substance, down the temperature gradient, till it escapes by radiation at the surface. The supply of heat must be kept up in some way; and one obvious process, as Helmholtz suggested long ago, is the slow contraction of the star. The work done by the gravitational forces in pulling the outer parts of the star toward the centre, reappears as heat produced by the compression, and maintains the star as a going concern. As the star contracts, its density must increase; and the pressure will increase too, for the various parts of the mass are nearer one another, and attract one another more strongly. When the star has shrunk to half its original diameter, the mean density will be eight times as great.

If the star, after contraction, continues to be "built on the same model," so to speak—that is, if the law according to which the density increases proportionally toward the centre remains the same, except for the altered scale of miles provided by the shortened radius, the density at any point, after contraction, will also be eight times the original density at the corresponding point (distant from the centre by the same fraction of the radius).

How will the pressures at the two points compare? The portion of the star nearer the centre than the point under consideration is compressed by the weight of the overlying portions. After the contraction, every part of these is twice as near the centre as before, and will, therefore, be

attracted four times more strongly. The whole compressive force will, therefore, be four times as great as at first; but the area over which this force is distributed will have shrunk to one-fourth of its former amount. Hence the pressure per unit of area will increase sixteenfold, as against an eight-fold increase of density. Applying the familiar laws of gases, we find that the temperature of the gas, after contraction, must be twice its original value in order that equilibrium shall still exist when the star has shrunk to half its former size. More generally, during the whole process of contraction, the temperatures at corresponding points will be inversely proportional to the star's radius—so long, indeed, as the star continues to be built on the same model, and the simple gas laws hold good. This proportion was first proved by Lane of Washington, in 1870, and is known as Lane's Law.

It appears at first sight paradoxical that a star may grow hotter by losing heat; but the difficulty disappears when it is realized that the heat produced by the contraction exceeds the amount which is required to raise the temperature of the mass to the extent demanded by Lane's Law. The remainder is available for radiation, and it is only as it is gradually lost into space that the process of contraction can take place. The manner in which the surface temperature of a star, which determines its color and spectral type, will vary as it contracts is somewhat different. As has already been shown, the light from the far interior of a star stands no chance of getting out to the surface, but practically all of it will be scattered away by the gases through which it passes, and remain inside the star. Light can only reach us directly from a relatively shallow layer close to the surface, and it is a certain sort of average of the temperatures throughout this layer that

gives the effective surface temperature. As the density of the star varies, the depth of this layer will alter, and in such a way that it always contains the same number of tons of material per square foot, since it is upon this quantity that the amount of scattering of light passing through the layer depends. As the star contracts, the total quantity of matter in this superficial radiating layer will therefore diminish proportionally to the surface area; that is, the radiating layer will form an ever decreasing part of the whole mass of the star, and its depth will be a smaller fraction of the star's radius. If the depth were a fixed fraction of the radius, we could apply the law of corresponding points and say that the temperature would vary inversely as the radius; but, in fact, after contraction the new radiating layer will form only the upper portion of the layer which "corresponds" to the old radiating layer, and its average temperature will be lower than that of the "corresponding" layer. On any reasonable assumptions regarding the way in which the temperature varies in the outer part of the star, it is found that the effective temperature of the surface will increase as it contracts, but much more slowly than the central temperature.

All these conclusions are based upon the fundamental assumption that the simple gas laws hold good throughout the star. This may safely be assumed if the density is low—say not more than twenty times that of air—but when the density begins to approach that of water, it will certainly be very far from the truth. As the density increases, the compressibility diminishes, so that, at the same temperature, it takes a greater increase of pressure to produce a further increase of density than would be necessary in a perfect gas. In other words, the material is better able than a perfect gas to stand up under pressure. Hence,

referring to the argument by which Lane's Law was proved, we see that a smaller increase of temperature than is demanded by this law will enable it to meet the changing conditions resulting from contraction. Indeed, a point will in time be reached, when no further rise of temperature at all is needed, the decreased compressibility of the dense gas taking the whole load. Beyond this the increased pressure due to contraction acting alone will be insufficient to produce the necessary increase in density, and a fall in temperature must complete the adjustment.

We see, therefore, that a sphere of real gas, contracting under its own gravitation, will follow Lane's Law only while its density is small. As it contracts further its temperature will rise more slowly than this law indicates, reach a maximum, and then gradually diminish. During this long process, the model upon which the mass is built will itself gradually change—the increase of density toward the centre diminishing—but this will not alter the general character of the phenomena. We may at least say with confidence that the surface temperature, as well as that in the interior, will reach a maximum and then diminish, until at last the mass will shrink nearly to the greatest density which it can possibly attain, and end by cooling off almost like a solid body. During the early stages, while the temperature is rising, the body will be of large diameter. As it contracts its surface will diminish, but its surface brightness will increase, so that the amount of light which it gives out will not change much. It will, however, grow whiter as it gets hotter, until it reaches its maximum attainable temperature. By this time it will be much smaller in diameter than at the start, but only a little fainter. But after it begins to fall in temperature, while still contracting, the situation is different. There are now fewer square

miles in its surface, and less light given out per square mile, so that its light will fall off rapidly, and it will grow fainter and redder until at last it disappears.

During its history, therefore, it will pass through any surface temperature lower than the maximum twice—once when of large diameter, low density, great luminosity and rising temperature, and again when its diameter is small, density high, luminosity low, and temperature falling. It is obvious that these contrasted groups of characteristics are exactly those which differentiate the giant and dwarf stars. The theoretical and observed pictures, indeed, agree not merely in their general outlines, but in every detail. For example, the lower the temperature selected for study, the greater will be the theoretical difference between the groups of stars of rising and falling temperature, and the greater is the actual difference between the giant and dwarf stars. The approximate equality in brightness among the giant stars of the various spectral classes, and the great differences among the dwarfs, find also a complete explanation.

Stars of large mass, as can easily be shown, should attain a greater maximum temperature than those whose mass is smaller, and should be more luminous than the latter, for the same surface temperature, especially in the giant stages. The great masses and luminosities of the B-stars are thus accounted for. The are not massive because they are hot, but hot because they are massive. Lesser masses never attain the B stage of temperature, but stop at A; and still smaller ones may not get beyond Class F, or even G. As we go down the spectral series, therefore, we are continually adding to our list stars of mass too small to get into any of our earlier groups at all—so it is no wonder that the average mass decreases for the red-

der stars. The fact that the masses of the giants average high, whatever their spectral type, is probably an effect of observational selection. We have picked then from a list of naked-eye stars, and hence from one in which the brighter stars have an egregious preference, and it has already been seen that, in these stages, great brightness means large mass.

A more searching test is found in the densities of stars of the various sorts; for here we can make our comparison quantitative instead of merely qualitative. The stars of increasing temperature should have densities at which the simple gas laws can be trusted to apply, at least approximately; the dwarfs should be so dense that we can be sure that these laws fail of application; while the hottest stars should have an intermediate density corresponding to the region in which the gas laws are strikingly at work. From a general knowledge of the properties of matter, we can say with certainty that a density less than ten times that of air falls in the first class, one greater than that of water in the second, while the "twilight zone" between corresponds to densities in the neighborhood of one-tenth to one-quarter that of water, and perhaps a little higher. Now we have already seen that the redder giant stars are less dense than air—the whiter ones being probably from ten to fifty times denser; that the average density of the A-stars is one-fifth that of the Sun, or one-third that of water, while their individual densities range from about fifty times that of air to that of water, and that the dwarf stars have densities running from about that of water up to four or five times as much. The agreement is perfect throughout, and there can be no remaining doubt that the proposed physical model represents what actually happens in the stars.

This theory of stellar evolution was first propounded by Sir Norman Lockyer who outlines clearly the physical processes involved. His criteria for distinguishing between stars of rising and falling temperature were spectroscopic, and chosen in a rather arbitrary way, with little explanation (though they were not very far from anticipating Adams' later discovery), and his views failed of general acceptance. It fell to the speaker's lot, some years later, to revive the theory, and point out the importance of the absolute magnitudes, which, indeed, furnish the key to the whole problem. This invaluable aid was not available when Lockyer began his work—for in those days little indeed was known of stellar parallaxes—so that it is not surprising that his individual assignments of stars to the classes of rising and falling temperature are often erroneous. With the wealth of material now available, it is an easy matter to point out stars in every successive stage of evolution, and to assign the large majority of those for which we have data to their place in its sequence. Mention should again be made, however, of the few, faint, but perplexing white stars of low luminosity. These do not fit into the scheme at all, and they present such an extraordinary combination of high temperature, small luminosity and considerable mass that it is very difficult to form any consistent idea of the physical conditions which exist on their surfaces. There are indeed more worlds for theory to conquer—and some of them look as if it would take hard fighting.

But there are other ways in which our knowledge of the properties of matter may be applied to the stars. A simple calculation shows that the gravitational pressure at the centre of the Sun must be something like a hundred million tons per square inch. The pressures in other dwarf

stars are of the same order of magnitude. Those in giant stars are smaller, but are usually measurable in thousands of tons per square inch, even when the density cannot be many times greater than that of air. To withstand such a pressure, at this density, the gas must have a temperature of many millions of degrees.

What can we say of the properties which matter would exhibit at these temperatures? Twenty years ago, the only answer would have been, "Very little"; but now, with our knowledge of atomic structure, we can say a good deal. The extreme violence of the collisions between the atoms would knock off all the electrons of the outer shells, and keep them off. The lighter atoms—perhaps as far as sodium or even beyond—would lose all their electrons, and be reduced to bare nuclei. The heavier ones would retain their innermost one or two rings or shells of electrons, but lose the outer ones, which contain a considerable majority of the whole number of electrons originally present. We can be certain, however, that the nuclei themselves would emerge quite unscathed from these collisions, and that if an isolated nucleus, or the battered fragment of a heavier atom, had a brief interval of relative quiet, it would begin to pick up electrons again from those which passed by slowly enough, and to reconstitute the atomic structure. Could we remove a portion of the matter in this strange state and let it cool, the familiar atoms would thereupon rebuild themselves, bit by bit, and at the end they would be the same as ever.

The principal differences at the high temperature, from our present standpoint are: first, there would be a vast multitude of free electrons flying about, as well as the far heavier atomic nuclei, so that the average "molecular weight"—in determining which every free-moving particle

in the gas counts as much as any other—would be much diminished. Secondly, the gas at this temperature would emit a tremendous flood of radiation, most of it of such short wave-length that it would resemble X-rays rather than ordinary light. This radiation would not go very far before it was scattered in all directions by the electrons, or absorbed in detaching some fast-knit electron from the remnant of an atom, only to be re-emitted when recombination took place. In either case the energy would be relayed back and forth from atom to atom, now in this direction, again in that, until in the lapse of ages it leaked gradually outward to the cooler parts of the star, on its way to the surface.

Jeans, was, I believe, the first to call attention to this extraordinary state of things, and Schwarzschild to point out the fundamental importance of the exchange of radiation in determining the conditions of equilibrium within a star; but the general solution of the problem came later, from Eddington, who was the first to appreciate one of its most fundamental features.

The flood of entrapped radiation, in its attempts to escape, exerts a pressure outward in all directions, just as a compressed gas would do. The existence of this radiation pressure was pointed out long ago by Maxwell's theory of light. With any light obtainable on Earth, even full sunlight, it is so minute that apparatus of the most delicate sort is required to indicate its existence; but at the temperatures which prevail inside the stars, it may amount (as Eddington pointed out) to hundreds of tons per square inch, and be an important factor in preventing the collapse of the star's interior under the weight of the outer parts. Indeed, under some conditions, it may do more than the gas-pressure due to the motions of the

atoms and electrons, huge as the latter is. Following this lead, and working out the laws of flow of energy outward down the temperature gradient, he showed that certain simple and probable assumptions about the opacity of the medium led to the conclusion that, all through the star, the gravitational pressure would be proportional to the fourth power of the temperature, and that the shares of this pressure which were sustained by the gas-pressure and the radiation pressure would be everywhere in the same ratio. These conditions, combined with the law of gravitation and the gas laws, suffice to determine completely the model upon which the star is built, and to tell us practically all that we need to know about it.

For the case where the simple gas laws hold, the mathematical work had already been done by Emden, who found that the outer regions of the star were of very low density, while there was a rapid concentration toward the centre, where the density reaches fifty-four times the mean density. The central temperature of such a star obeys Lane's Law, while the surface brightness varies inversely in the square root of the radius. This means that the whole amount of energy radiated from the star's surface will be independent of its size—the increase in surface brightness and decrease in area, as it contracts, balancing one another exactly. The amount of the star's radiation depends upon the opacity of its material—diminishing as this increases—and is also proportional to the ratio which the radiation pressure bears to the total pressure at any part inside the star. This ratio increases rapidly with the star's mass, and the brightness should do the same.

These conclusions form a theory of giant stars. To extend it to dwarf stars Eddington repeated his calculations, taking into account the manner in which the com-

pressibility of a gas decreases with increasing density, and obtained a theoretical table which represents the way in which the absolute magnitude and temperature of a star should depend upon its mass and density throughout the whole range of these quantities. This table reproduces the actual characteristics of the dwarf stars, and those of maximum temperature, as well as the giants, with a fidelity which is almost uncanny, and far more than justifies its author's modest claim that the theory upon which it is based "gives a fair approximation to the facts."

But Eddington's theory goes beyond this. It actually shows us why the masses of the stars are so much alike, and why they are of their actual order of magnitude. If β is the fraction of the whole pressure within the star which is balanced by the gas-pressure, leaving the fraction $1-\beta$ for the radiation pressure, he derives by reasoning of a very general character, the equation

$$\frac{1-\beta}{\beta^4}=4.6\times10^{-68}M^2$$

where M is the mass of the star in grams. The extraordinarily small numerical coefficient depends only upon a few very fundamental natural constants—the gravitational constant, the quantum, and the average mass of one of the "molecules" in the star (including in this term atoms, nuclei and free electrons). The numerical value here given depends on the assumption that the last quantity is 2.8 times the mass of a hydrogen atom, an estimate which must be nearly correct if the atoms are dissociated into nuclei and electrons to the degree which has been described.

Now, following Eddington's argument, we may imagine a set of spheres of gas, each isolated in space and in equilibrium under its own gravitation and radiation, the

first mass of 10 grams, the next 100 grams, the third 1,000 grams, and so on. Then, by means of his equation, we find that the proportion which the radiation pressure bears to the whole will be quite negligible in all the spheres up to number 32, will increase rapidly for numbers 33 and 34; while for sphere 35 and all those beyond it the radiation pressure will be the dominant partner, leaving little for the gas-pressure to do.

Upon this long line of spheres, therefore, we find a small region in which a certain natural factor changes from an insignificant to a controlling role. On general physical principles, therefore, as Eddington puts it, we would expect "something to happen" in this critical interval, and "what happens is the stars." It is only when the radiation pressure and the gas pressure share the gravitational food that we get anything that can fairly be called a star. Smaller masses do not give out light enough to make them visible at interstellar distances, while the great ones, in which the radiation pressure is almost sufficient to counteract gravitation, would be in an almost unstable condition, so that a small disturbance, such as might be produced by a moderate rotation, would cause them to break up into parts. Hence smaller masses do not shine, and bigger ones break up, and only those in the critical intervening range of mass remain as luminous stars. We have seen that this should occur for masses comparable with those of spheres 33 and 34 of the series. Now the first of these is of half the mass of the Sun, and the second has five times the Sun's mass, so that the actual masses of the stars fall very exactly into the range indicated by the theory. Since the constants of this theory are derived from those which are the most fundamental in modern physics, we may truthfully say that the masses, and hence the sizes

and brightness, of the stars are determined directly by the fundamental properties of the very atoms of which they are composed. It may be shown, for example, from Eddington's equation, that a mass of gas will shine as a giant star when, and only when, the ratio of the diameter of the star to the average distance between the atoms which compose it is about twenty times the ratio between the charge of an electron and the average mass of an atom (provided that this mass is measured, not in the ordinary way, but, as in the electrical case, by its power of attracting a similar mass at a given distance). The latter ratio is very large, about 4×10^{17}, so that the number of atoms in the star is enormous, and the star itself a very large mass.

One of the most impressive consequences of the whole theory is that the masses of the stars are determined by the interplay of the two forces, gravitation and radiation pressure, which, among all those in nature, are so feeble, under the conditions of ordinary experiment, that it taxes the skill of the experimenter to build an apparatus delicate enough to measure the effects of either one. Were we confined to experiments in enclosed laboratories, isolated in space, without the Earth's attraction to prove to us the existence of gravitation, it would probably have been long before the very existence of either of these forces would have been suspected; yet these forces, and these alone, when working on the grand scale, are powerful enough to shape the stars.

One question still remains. How long a time is required for this sequence of evolutionary changes? What is the life of a star? Here, again, the answer which we would now give depends upon knowledge which has come within the last decade or two. We have, even now, no direct

evidence regarding the age of the stars, or the Sun; but we have information about the age of the Earth that has magnified our conception of the duration of the universe in time, in as startling a fashion as the study of the globular clusters has enlarged our idea of its extension in space.

The new method of measuring times is really very simple. Uranium is radioactive, and slowly "decays." One by one its atoms eject a part of their nuclei, and change into atoms of a different element. These again break up, and so on through a long and wonderful series of transformations, in which radium is one step. The particle ejected from the nucleus is sometimes an electron, but oftener an alpha particle, identical with the nucleus of a helium atom. Finally, at the end of the list, there remains a stable atom of lead—but not of ordinary lead, for its atomic weight is 206, instead of 207 as usual. In the course of ages, this radio-lead must accumulate in all uranium minerals. The rate of accumulation is accurately known, from a study of radio-active phenomena and we can be sure that the weight of lead produced in a millon years is 1/8000 of that of the uranium which is present. By determining the percentages of uranium and lead now present in a mineral, and applying this principle, we can find out how old the mineral is—provided, of course, that it contained no lead, when it was originally formed by crystallization in the molten rock mass. Such primitive lead would, however, be ordinary lead, of higher atomic weight, and a determination of the atomic weight of the lead derived from one specimen will enable us to tell how much of it was there when the mineral formed, and how much has been produced by radio-activity since this time.

In this way reliable values can be obtained for the ages of various minerals, and the dates of the eruption of the

rocks in which they occur. The latter can often be defined in geological terms, and hence we can date the various geologic periods, finding a good general agreement with the geological order of succession. The oldest minerals so far studied are found in rocks of Middle Pre-Cambrian age. Specimens from Europe, Africa and America agree in giving ages of between a thousand and twelve hundred millions of years. These individual crystals have been in the rocks for all this time. The Earth, as a planet, must be older. The speaker, from consideration of the whole amount of uranium and lead in the Earth's crust, showed last year that its age is apparently less than eight billions of years, and probably something like four billions. If, as seems most probable, the planets were produced by eruptions from the Sun, under the tidal influence of a passing star, the Sun itself must have been already formed at that remote epoch.

But we may go further. Life already existed on the Earth in Pre-Cambrian times, and it is a moderate estimate to say that the process of organic evolution has lasted for a billion years. During all this time the Sun can never have been one stellar magnitude brighter or fainter than it is now, for in the first case, its heat would have raised the oceans to the boiling point, and in the second, they would have frozen solid—and either of these catastrophes would have put an end to evolution and to all terrestrial life. Now the Sun is a typical dwarf star, and there is good reason to believe that it is now well advanced in cooling and was once much brighter and hotter than it is now—of Class F, at least, though perhaps not of Class A. At such a time it must have been at least two magnitudes brighter than at present. Yet in the whole of geological time it has probably decreased half a magnitude or less.

We may, therefore, say, with considerable confidence, that the life of the Sun, and doubtless also of the stars in general, must extend over many billions of years.

But here we meet with a serious difficulty. We know the rate at which the Sun is radiating energy to the earth, and, from consideration of the way in which the Earth in turn radiates this energy into space, we can be sure that the Sun is also sending out an equal amount of heat into space in every direction. The total output is so great that it would exhaust the whole huge fund of energy, which would be made available by the Sun's contraction from an indefinitely extended size, in about twenty million years, as Lord Kelvin showed long ago. When we allow for the fact that some of this heat is still stored in the Sun's interior, and that it was probably much brighter in its earlier stages of evolution than at present, we see that, if gravitational energy alone was available as the source of its radiation, the Sun's past life as a star must have occupied but a very few million years. In view of the geological and radio-active evidence, there seems to be no escape from the conclusion that the Sun must have some other, and far greater, store of internal energy upon which to draw.

Further evidence in favor of this view has been found by Eddington in the behaviour of the star Delta Cephei. This is a typical giant star, about eight hundred times as bright as the Sun. Eddington has given good reason to believe that the cause of its variation in light is a periodic expansion and contraction of the whole star by about ten per cent. on each side of the mean. The period of this change would depend on the density of the star, and diminish if this increased. Hence, if the mean diameter was gradually contracting, the period should shorten. Eddington calculates that, if the radiation is

supplied by gravitational contraction alone, the period should decrease by about forty seconds per year. The observations, which cover more than a century, show indeed a decrease of period, but at the rate of about a second in twelve years—five hundred times slower than the previous theory would demand. Here again we have evidence that the rate of stellar evolution—in a giant star this time—is many hundreds of times slower than it would be if there was not some internal store of energy to draw upon.

It is certain that no corresponding evolution of heat from any source occurs within the Earth, and we must therefore suppose that energy from the "unknown source" becomes available only at exceedingly high temperatures, such as prevail inside the stars. But if this is the case, and a star, in contracting, gets hot enough inside to start this process going, why does this not make the interior still hotter, and so cause a still more rapid transformation of the unknown energy into heat, till the process ends in an explosion on a colossal scale? I mulled over this idea for a couple of years before I saw the simple answer. If heat energy is supplied to the interior of a giant star, the star will have to expand, and if it expands, it must grow cooler. The process is the exact reverse of that by which contraction makes the star hotter, and at the same time compels the escape of heat from the surface into space. Hence, if too much heat is supplied from the unknown source, the star will expand and cool, shutting off farther supplies. It is easy to see that we have here a self-regulating process, which, in the long run, will automatically adjust the supply of heat in the interior so that it just makes up the loss due to leakage toward the surface and radiation into space. In the

short run, we might find alternate over-production, leading to expansion of the star and cooling, and under-production, permitting contraction and heating; and oscillations of just this sort appear to happen in the Cepheid variables. Though the star may thus be kept shining for a very long time, it cannot go on forever, for the store of internal energy, however vast, must be finite, and will gradually be used up. As this happens, the star will contract, although very slowly, and ultimately pass through the various giant and dwarf stages, in substantially the manner which was described earlier.

Such a store of available energy will account for the facts; but how shall we attempt to account for the store of energy itself? One thing is clear at the start. The only places small enough to contain so huge an accumulation are the nuclei of the atoms. I say "small" advisedly, for it is only when the constituent parts of which the atoms are built come exceedingly close together that the forces between them can become great enough to account for their possession of such an amount of energy. Radio-active energy, which comes from atomic nuclei, represents indeed one such gigantic store. But the amount of energy which must once have been stored in each gram of the Sun's mass, to account for its past radiation of heat, is even greater than that contained in uranium. We cannot do more than guess where it may have been hidden; but one very recent piece of work affords a possible clue.

Aston, in one of the brilliant researches which we have learned to associate with the Cavandish Laboratory at Cambridge, has invented a beautiful apparatus which sorts atoms, by giving them electrical charges and shooting them through a vacuum under the influence of electric and mag-

netic fields. The resulting forces deflect atoms of different weights in different directions, and bring each kind to a separate focus upon a photographic plate, producing images when the plate is developed. By measuring these plates the atomic weights may be determined; and Aston has found that, in every case but one, the true atomic weights are exact integers, within the accuracy of measurement, which is about one part in a thousand. When the chemist finds an atomic weight which is not an integer, such as 35.46 for chlorine, this is really the average for two different kinds of atoms of the same chemical properties, but different weights, both of which are integers—35 and 37 in this case. The one exception is hydrogen, for which the chemist's determination 1.008 is exactly confirmed.

Now it is more than a century since Prout suggested that, since the atomic weights are so nearly integers, the atoms themselves are built up out of simple units. We now transfer this idea to the atomic nuclei, which contain practically all the mass, and Aston's beautiful researches practically compel belief. The hydrogen nucleus, or "proton," is the lightest of all, and we would naturally look to it as the fundamental unit. Rutherford's success in knocking the nuclei of elements such as oxygen, nitrogen and sodium to bits, by collision with fast moving alpha particles, has furnished a definite proof that protons, and alpha particles as well, are actual constituents of these nuclei. Many nuclei must also contain electrons, which prevent the net electric charge from getting too high. It looks, for example, as if an alpha particle was built of four protons and two electrons, held together by forces of whose nature we are ignorant. This would give exactly the right electric charge; but the mass of the four

protons would be greater than that of the alpha particle by one part in 130. (The electrons weigh next to nothing.) This seems to spoil the explanation altogether, but an escape is found in that great resolver of otherwise intractable difficulties, the Principle of Relativity. According to this, all energy has mass, and all mass is equivalent to energy. The loss of mass in the formation of the alpha particle would mean that, in forming it, energy would be liberated, which would have to be put back into it again in order to separate the parts. The calculated amount of energy is so enormously great that it is not at all surprising that the alpha particle is so stable. Even in the collisions with other atomic nuclei which shatter the latter into fragments, the forces (which can be roughly calculated) are not nearly strong enough to disintegrate it.

We may now suppose that, in the interior of the stars, and by some process the details of which are still quite unknown, the atoms of hydrogen are taken apart, and the pieces—protons and electrons—built up into the nuclei of heavier atoms, with just enough electrons left over to build the outer parts of these. We cannot be sure, of course, that such a thing actually happens; but, if it does, the energy liberated will suffice for the present demands of astrophysics. If the Sun, for example, was originally all hydrogen, which was transformed in this fashion into other elements, the energy which would be set free as a by-product would keep it shining at the present rate for about 120 billions of years.

Such is our present conception of the stars, their distance, their age, their nature, and their life-history. In the grandeur of its sweep in space and time, and the beauty and simplicity of the relations which it discloses

between the greatest and the smallest things of which we know, it reveals as perhaps nothing else does, the majesty of the Order about us which we call Nature, and, as I believe, of that Power behind the Order, of which it is but a passing shadow.

HENRY NORRIS RUSSELL.

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